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Analyzing the Performance of Diversified Commodity Derivatives Portfolios in Brazil

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Abstract: *This paper seeks to investigate how we invest in commodity derivatives by comparing the performance of portfolios constructed using the mean-variance model, hierarchical risk parity, and the naive strategy with the stock market and the risk-free rate. For this purpose, daily closing future prices of ethanol, corn, cattle, gold, coffee, and soybeans traded in Brazil from 2015 to 2023 are considered, in addition to the rate of interbank deposit certificates (CDI or DI) and the performance of the theoretical portfolio. of shares in the Ibovespa index. The constructed portfolios are analyzed using the Sharpe Ratio, CVaR, Beta, and Maximum Drawdown metrics and the comparison of their returns. The results indicate that the portfolio constructed using the mean-variance model presented the lowest risk ratios and generated the highest return, followed by the hierarchical risk parity, the naive strategy, and the Ibovespa index. These findings are useful for scientific literature that investigates finance by bringing empirical evidence of Brazilian commodities to portfolio managers, investors, and other agents in the supply chain.*

Keywords: *Portfolio diversification, Investments in derivatives, Brazilian commodities, future prices.*

JEL Classification: *G00, G11, Q11, Q14.*

1. Introduction

Brazil is known worldwide for being a large producer of commodities. The emerging country is the world's largest producer of soy and ethanol, the second largest producer of beef, the third largest producer of corn, and has been the largest coffee producer in the world for more than 150 years. The Brazilian economy depends heavily on the agricultural sector, with 24.4% of the country's GDP coming from the sector while employing around a third of the total number of workers.

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Due to such economic importance, commodity prices can be used as a protection instrument in inflation cycles, which are very common in emerging countries and tend to transmit inflationary pressure due to supply and demand expectations (Souza, 2015). These prices can also present significant volatility and, to mitigate the possible risks posed by these variations, producers and economic agents in the supply chain see the negotiation of future contracts for these goods in the capital market as a plausible alternative.

Likewise, caused by the intensification of the financialization of commodities that occurred at the beginning of this century, investors and portfolio managers began to allocate more and more of their resources to these derivatives, seeking to reduce the exposure of their portfolios to risk. This motivation originates from the historically lower correlation between commodity prices and other assets traded in the capital market, such as shares, foreign exchange, and public bonds, for example.

For portfolio managers, there are two types of risk, systematic and unsystematic. Systematic or market risk cannot be eliminated by diversification because it affects the entire market, assets, and companies, such as wars, inflation, and recessions. Unsystematic risk is specific to a given asset or sector and is easily eliminated by diversifying the asset portfolio (Renner, 2010).

The work that inaugurated the scientific literature on finance was that of Harry Max Markowitz, in 1952 (Passos et al., 2022). Modern portfolio theory, with Markowitz's (1952) mean-variance model, influences all modern investment portfolios, serving as a basis for developing new forms of diversification and thus comparing their returns when considering the risks involved. It creates a portfolio optimization model that maximizes results while minimizing portfolio investment risks (Ma, Han, and Wang, 2021).

Based on the work of Markowitz (1952), which was developed from solid mathematical bases when analyzing risk and return and showing that portfolio risk does not depend exclusively on each asset, but on the covariance between isolated assets, this method changed the focus of individual analysis for the use of diversification. The author realized that, in an attempt to reduce risk, it is not enough to merely diversify among many assets, it is also important to invest in assets with a low correlation between their returns (Alves, 2015).

Thus, the present work seeks to investigate the performance of commodity derivatives as an investment strategy by constructing three commodity investment portfolios using the Markowitz mean-variance and Hierarchical Risk Parity (HRP) models, in addition to a naive strategy, and comparing its performance with the Brazilian capital market and with a risk-free asset. For this, the closing prices of futures contracts for the commodities ethanol, corn, cattle, gold, coffee, and soybeans traded on the Brazilian stock exchange from March 2015 to April 2023 are considered.

The performance of the theoretical portfolio of shares that forms the Ibovespa index is used as a thermometer of the Brazilian capital market and the rate of interbank deposit certificates (CDI or DI) as risk-free asset, as they are linked to Brazilian public bonds. Portfolio performance is measured using the Sharpe ratio, Beta, CVaR, volatility, and Maximum Drawdown.

The results indicate that the commodity investment portfolio constructed using the Markowitz model generated the highest return, followed by the portfolio constructed using the hierarchical risk parity (HRP) method, the naive strategy, and the Ibovespa index, used as a market benchmark. Risk measurements showed that the Markowitz model presented the lowest risks among the others, in addition to generating an accumulated return greater than the risk-free rate and pointing out that investments in commodity derivatives should be considered investment options.

Several studies have investigated the allocation of investments in commodities and portfolio diversification, bringing important contributions such as Akey (2006), Tang and Xiong (2012), Bessler and Wolff (2015), Jawadi and Ftiti (2017), Yan and Garcia (2017), Aït-Youcef (2019), Mercer (2009), Skiadopoulos (2012), Tuysuz (2013), Hu (2022), Lavaja et al. (2006), Natoli (2021), Gordon (2006), Belousova and Dorfleitner (2012), Shahzad et al. (2014), Skapa et al. (2013) and Batten et al. (2015).

These findings are useful for this literature that investigates finance and investments by bringing empirical evidence for the performance of investments in commodity derivatives in Brazil, for financial market agents, portfolio managers, and companies operating in the supply chain.

In addition to this introduction, the work is composed of three more sections, in which, in the second section, the database and methodologies that build portfolios and performance measures are presented, the third section presents and discusses the results and, finally, the fourth section concludes.

2. Methodology

2.1. Data

The database considered a time series of daily closing prices of futures contracts for six commodities traded on the Brazilian stock exchange, covering the period from December 1, 2015, to December 31, 2022. The commodities are ethanol, corn, cattle, gold, coffee, and soy. Furthermore, the daily closing rate of interbank deposit certificates (CDI or DI) - often used as a risk-free rate as they remain at the levels of Brazilian public bonds - and the theoretical portfolio of stocks that forms the Ibovespa index - used as a performance measure of the Brazilian stock market.

2.2. Portfolio Formation

The construction of Brazilian commodity portfolios is carried out through three strategies. The naive strategy, where all assets have the same investment weight, the Markowitz mean-variance method, which analyzes performance and assigns different weights to each asset, and the Hierarchical Risk Parity, which applies graph theory and machine learning to build a diversified portfolio based on the information contained in a covariance matrix.

2.2.1. Naive strategy

The naive strategy of portfolio diversification is used - where all assets have the same weight in the portfolio, with each commodity having the same proportion of values invested (Alves, 2015), such as:

$$P = {}^{1}\!/\!{}_{N}\,\# \tag{1}$$

2.2.2. Markowitz Model

Markowitz's (1952) Mean-Variance model is used to obtain the proportion of each asset in the capital allocation. It is worth remembering that the model does not take into account investors' risk preferences and risk-free assets, having only risky assets in the composition. With the assets and their appropriate investment proportions, it will be based on daily performance (Wang *et al.*, 2020). The model is taken as:

$$\min \frac{1}{2} \Sigma_{i=1}^{N} \Sigma_{j=1}^{N} \omega_i \omega_j \sigma_{ij} \tag{2}$$

Subject to $\Sigma_{i=1}^{N} \omega_i R_i = \sigma\rho$ and $\Sigma_{i=1}^{N} \omega_i = 1$. Given the target expected rate of return of portfolio μρ, find the portfolio strategy that minimizes σ_{ρ}^{2}.

2.2.3. Hierarchical Risk Parity

As shown by Da Costa (2022), De Prado (2016) introduz a abordagem Hierarchical Risk Parity (HRP). The HRP algorithm is based on three stages:

tree clustering, quasi-diagonalization, and recursive bisection. The first step involves dividing the assets into different clusters using the Hierarchical Tree Clustering algorithm. For two assets i and j, the correlation matrix is transformed into a correlation distance matrix D, according to Equation 3.

$$D(i,j) = \sqrt{0{,}5 \times (1 - \rho(i,j)} \tag{3}$$

Subsequently, the Euclidean distance between all columns is calculated in pairs, to find the distance matrix $\bar{D}$, described in Equation 4.

$$\bar{D}(i,j) = \sqrt{\sum_{k=1}^{N} \left(D(k,i) - D(k,j)\right)^2} \tag{4}$$

Given the distance matrix $\bar{D}$, a set of U clusters is constructed using a recursive approach. Therefore, the first cluster (i^*, j^*) is calculated according to Equation 5.

$$U[1] = \arg\min_{i,j} \bar{D}(i,j) \tag{5}$$

After calculating the first cluster, the matrix $\bar{D}$ is updated by calculating the distances of the other assets in the U[1] cluster using single-link clustering. The aim is to recursively combine the assets in the portfolio into clusters and update the distance matrix until only a single cluster is left Therefore, for any asset i outside the cluster, the distance to the newly formed cluster is found following Equation 6.

$$\bar{D}(i,U[1]) = \min\left(\bar{D}(i,i^*), \bar{D}(i,j^*)\right) \tag{6}$$

The second stage rearranges the rows and columns of the covariance matrix so that the largest values lie along the diagonal. As well described by De Prado (2016), the quasi-diagonalization step causes similar investments to be placed together in the covariance matrix, and different investments to be placed farther apart.

The final recursive bisection step involves assigning real portfolio weights to assets. The algorithm exploits the portfolio property that the inverse variance allocation is optimal for a diagonal covariance matrix.

2.3. Methods for Measuring Performance

The Sharpe Index, Maximum Drawdown (MD), Conditional Value at Risk (CVaR), and Beta methods are used to evaluate constructed portfolios, as in Bessler *et al.* (2017).

2.3.1. Sharpe Ratio

The Sharpe Ratio is widely used in investment evaluation, as it can numerically determine the risk-return of a portfolio of assets. The index shows us profitability and whether it is compatible with the risk it is exposed to. Therefore, we can compare their respective results and determine a better portfolio option (Brum, 2008). As in Equation 7.

$$S = \frac{\overline{r}_S - r_f}{\hat{\sigma}_S} \tag{7}$$

Where,

S is the Sharpe ratio;

$\overline{r}_S$ is the average return on the portfolio;

r_f is the rate of return on the risk-free asset or rate of return on the benchmark index; and

$\hat{\sigma}_S$ is the standard deviation (volatility) of the investment.

2.3.2. Maximum Drawdown (MD)

This indicator seeks to highlight the biggest drop that has ever occurred in the asset from a high point to the minimum point in a historical series (Da Costa, 2022). As in Equation 8.

$$MD_S = Max_{s,t^* \in (0,T)}\left[Max_{s,t\epsilon(0,t^*)}\left(\frac{P_{s,t} - P_{s,t^*}}{P_{s,t}} \right) \right] \tag{8}$$

Where,

$P_{s,t}$ is the price of the portfolio generated by the strategy s in time t; and

P_{s,t^*} is the price of the portfolio generated by the strategy s in time t^*, when the portfolio is sold.

2.3.3. Conditional Value at Risk (CVaR)

Value at Risk is a unique and summarized statistical measure of possible portfolio losses, it results from normal market movements (Linsmeier & Pearson, 2000). According to Da Costa (2022), CVaR complements Value at Risk, as it tries to address VAR's deficiencies. As in Equation 9.

$$CV\alpha R = \frac{1}{1-c}\int_{-1}^{V\alpha R} xp(x)dx \tag{9}$$

Where,

$p(x)dx$ is the probability density of obtaining a return with a value of x;

c denotes the cutoff point of the distribution.

2.3.4. Beta

The beta coefficient, β, is used to measure portfolio volatility about systematic market risk. It measures the sensitivity of the portfolio, seeking to understand whether it moves in the direction of the market, and showing information on whether there is a relationship between the portfolio and the rest of the market (Da Costa, 2022). As in equation 10.

$$\beta = \frac{Covariance(R_p, R_m)}{Variance(R_m)} \tag{10}$$

Where,

R_p is the return on an individual portfolio P; and

R_M represents the return on the market portfolio.

3. Results

Table 1 presents the statistics of the daily returns of the six commodities considered.

Table 1: Estatística descritiva dos retornos

Asset	*Average*	*Srd. Dev.*	*Minimum*	*25% percent*	*50% percent*	*75% percent*	*Maximum*
Ethanol	0.059%	1.764%	-16.995%	0.000%	0.000%	0.296%	21.629%
Corn	0.052%	1.541%	-14.510%	-0.670%	0.025%	0.775%	13.814%
Cattle	0.035%	0.868%	-0.077%	-0.246%	0.000%	0.299%	6.744%
Gold	0.061%	1.419%	-6.467%	-0.750%	0.000%	0.786%	11.575%
Coffee	0.041%	2.139%	-8.992%	-1.251%	0.000%	1.204%	11.947%
Soybeans	0.027%	1.271%	-7.581%	-0.676%	0.000%	0.688%	5.612%

Source: Elaborared by authors.

Through Table 1, we can see that all assets have a positive average daily return, with the highest return being that of gold at 0.061%. Table 2 presents the correlation between commodities.

With Table 2, it is possible to observe that there are varied correlations, with gold and soybeans being the commodities with the lowest correlation, at -0.129, and corn and soybeans with the highest correlation, at 0.173. If there are values that are little related between them, the possibilities for

Table 2: Correlation Matrix

Pearson's correlation	*Ethanol*	*Corn*	*Cattle*	*Gold*	*Coffee*	*Soybeans*
Ethanol	1.000	0.003	0.046	-0.035	0.002	0.039
Corn		1.000	0.094	0.096	0.020	0.173
Cattle			1.000	0.006	0.028	0.045
Gold				1.000	-0.128	-0.129
Coffee					1.000	0.176
Soybeans						1.000

Source: Elaborated by authors.

diversification can be increased. Table 3 presents the results of the performance evaluation of commodity portfolios constructed based on the Markowitz model, the Hierarchical Risk Partition, the naive strategy, and the theoretical framework of the Ibovespa Index.

Table 3: Avaliação de Performance

Variable	*Markowitz*	*HRP*	*Ingênua*	*IBOV*
Annualized return	10.303%	9.715%	9.464%	9.270%
Sharpe ratio	0.99	0.95	0.89	0.48
Drawdown maxim.	-20.75%	-19.28%	-21.46%	-46.81%
CVaR	-1.574%	-1.701%	-1.652%	-3.791%
Beta	-0.06	-0.03	-0.01	1.00
Accumulated return	119.30%	110.11%	106.30%	103.43%
Annual volatility	10.47%	10.27%	10.78%	25.59%

Source: Elaborated by authors.

From Table 3, we can verify the performance of portfolio diversification strategies for commodities and a market benchmark, which is the theoretical portfolio of the Ibovespa index. Regarding accumulated return, the portfolio generated through the Markowitz model performed better than the portfolio generated through hierarchical risk parity, even though it is a more recent model and built using machine learning algorithms.

Both portfolio diversification strategies obtained more effective results to generate superior returns when compared to the naive strategy and the Ibovespa Index, which obtained similar returns. The portfolios generated from the Markowitz and hierarchical risk parity models presented higher Sharpe ratios, being 0.99 and 0.95, respectively. This indicates that both generated better risk-adjusted returns compared to the returns of the naive strategy and the Ibovespa index.

Regarding the Maximum Drawdown, which measures the maximum drop in the portfolio, the portfolios generated from the Markowitz and hierarchical risk parity models proved to have the lowest impacts in periods of asset devaluation, -20.75% and -19.28 % respectively. However, it is important to highlight that the portfolio built based on the naive strategy obtained a very close result, with -21.46%, while the Ibovespa index used as a market benchmark obtained the worst results, with -46.81%, indicating that commodities performed better in periods of decline than the Ibovespa index.

The CVaR showed a lower expected loss for Markowitz's portfolio, with -1.574%, while the hierarchical risk parity and the naive strategy performed closely, with -1.701% and -1.652, respectively. Regarding annual volatility, the portfolio using the hierarchical risk parity diversification strategy had the lowest rate, with 10.27%, followed by Markowitz, with 10.47%, and the naive strategy with 10.78%, while the benchmark had the highest annual volatility, with 25.59%.

Among the portfolios constructed, the one that generated the best-accumulated return was the one generated with the Markowitz model, with 119.30%, followed by the portfolio generated by the hierarchical risk parity, with 110.11%, while the portfolio that used the naive and the Ibovespa index returned 106.30% and 103.3%, respectively.

As the commodity investment portfolio constructed based on the Markowitz model was the one that presented the best performance, for illustration purposes, an initial investment of R$100,000.00 is simulated. Markowitz's portfolio performed better than the Ibovespa index, with an accumulated return of R$219,365.70 compared to R$203,428.58. In addition to the return, Markowitz's portfolio had much less annual volatility, as seen in Table 3, being 10.47%, while that of the Ibovespa index was 25.59%. If compared to the Brazilian risk-free asset, the interbank deposit certificate, Markowitz's portfolio also proved to be superior, with this accumulated asset in the period generating only R$ 197,126.18.

Figure 1 presents the accumulated return of the commodities portfolio constructed using the Markowitz model and compares it with that of the Ibovespa index and the interbank deposit rate - or DI.

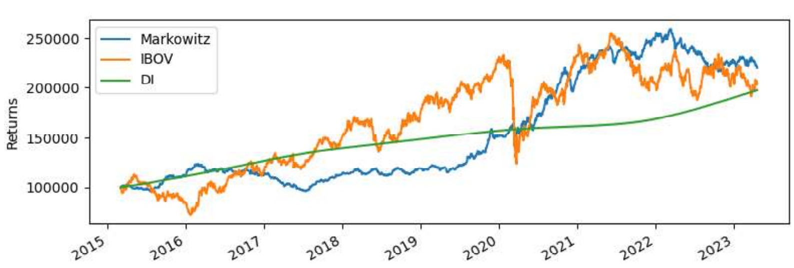

Figure 1: Markowitz's cumulative portfolio returns

Source: Elaborated by authors.

Continuing with the simulation, the portfolio using the hierarchical risk parity method recorded an accumulated return of R$ 210,110.47, compared to R$ 203,428.58 for the Ibovespa index and R$ 197,126.18 for the rate on interbank deposit certificates – CDI or DI. Figure 2 compares the accumulated returns of the commodity portfolio constructed using the hierarchical risk parity method with the benchmark and the risk-free rate.

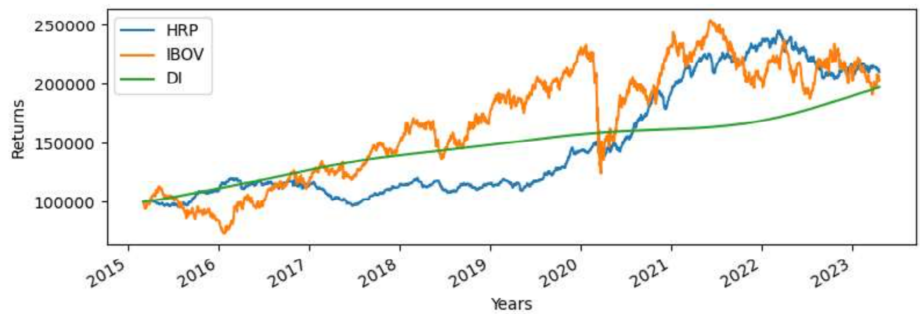

Figure 2: Hierarchical risk parity cumulative portfolio returns

Source: Elaborated by authors.

With the simulation, it is possible to see that the commodity investment portfolio constructed through the naive strategy obtained an accumulated return of R$ 206,296.98, while the Ibovespa index obtained an accumulated return of R$ 203,428.58 and the rate of interbank deposit certificates – CDI or DI - obtained an accumulated return in the period of R$ 197,126.18. Figure 3 compares the Naive strategy with the benchmark and the risk-free asset.

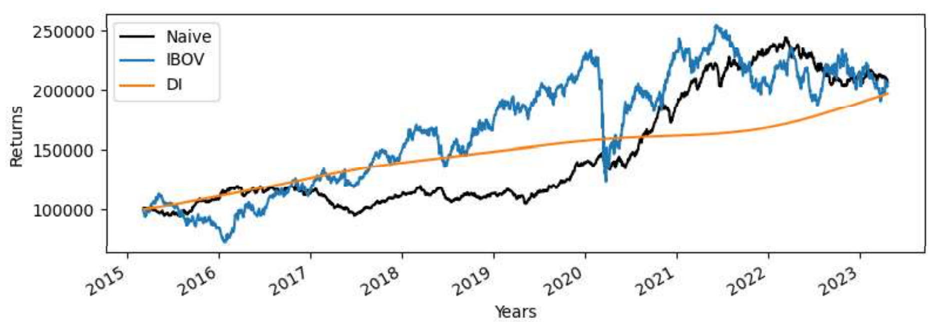

Figure 3: Naive strategy cumulative portfolio returns

Source: Elaborated by authors.

When comparing the accumulated returns of the three investment portfolios in Brazilian commodity derivatives, it is possible to see how similar they behaved over the period. However, the portfolio constructed using the Markowitz model presented superior returns, followed by the portfolio constructed using hierarchical risk parity and the portfolio that used a naive strategy. With the simulation, the Markowitz portfolio obtained a value of at the end of the period R$ 219,365.70, the hierarchical risk parity portfolio generated an accumulated return of R$ 210,110.47, and the naive strategy of R$ 206,296.98. Figure 4 presents the comparison between the three commodity portfolios.

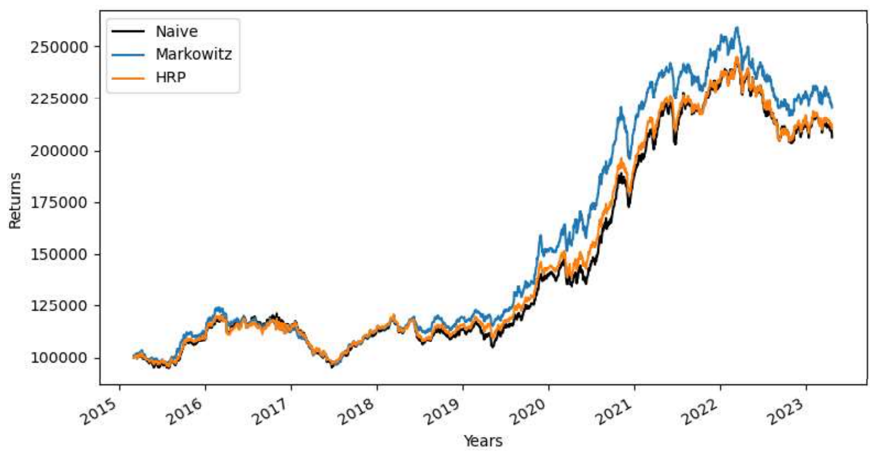

Figure 4: Commodities' cumulative portfolio returns

Source: Elaborated by authors.

Figure 5 highlights the ten biggest drawdowns for each of the portfolios in the period between 2015 and 2023.

As can be seen in Table 1 and Figure 5, the portfolio constructed using hierarchical risk parity had the smallest maximum drop, with 19.28%, followed by the Markowitz portfolio, with a 20.75% maximum drop. The naive strategy generated a maximum drawdown of 21.46% in the same period, while the theoretical stock portfolio of the Ibovespa index used as a benchmark had a maximum drop of 46.81%. Thus, it is noticeable that the use of Markowitz and hierarchical risk parity portfolio diversification methods improves return and risk in general, whether with smaller drops or with higher returns.

4. Final Considerations

The present work sought to investigate the allocation of investments in commodity derivatives by comparing the performance of portfolios constructed using the Markowitz mean-variance and hierarchical risk parity diversification models, in addition to a portfolio that used a naive strategy. For this, daily closing prices of futures contracts for the commodities ethanol,

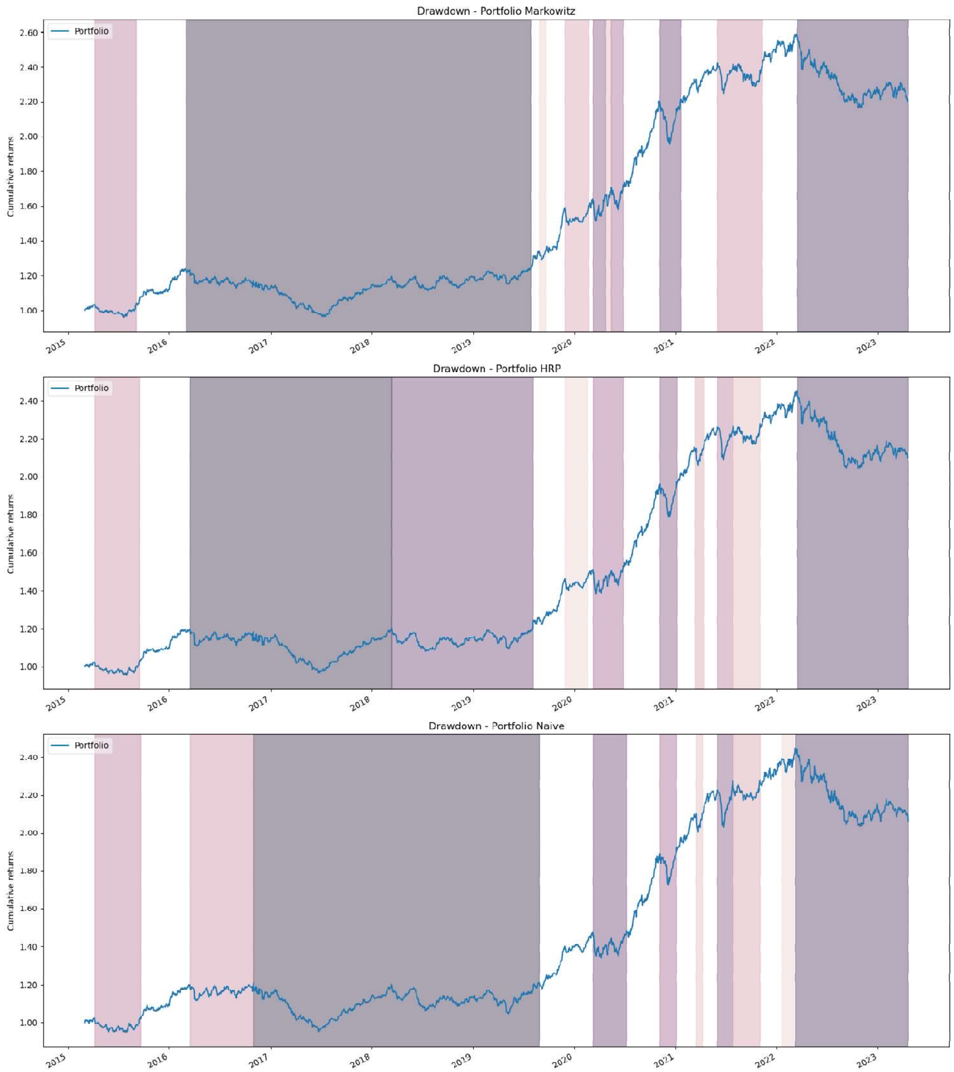

Figure 5: Top ten commodity portfolio drawdowns

Source: Elaborated by authors.

corn, cattle, gold, coffee, and soybeans traded on the Brazilian stock exchange from March 2015 to April 2023 were considered.

For purposes of comparison with the Brazilian capital market and with risk-free assets, the performance of the theoretical portfolio of shares that form the Ibovespa index and the rate of interbank deposit certificates (CDI or DI), frequently used as a rate, are also considered. risk-free in Brazil as they are linked to Brazilian public bonds. To measure portfolio performance, the Sharpe ratio, Beta, CVaR, volatility, and Maximum Drawdown are considered.

Risk measures suggest that the portfolio using the Markowitz model had the lowest risk ratios relative to its peers. The results indicate that the portfolio constructed using the Markowitz model generated the highest return, followed by the portfolio constructed using the Hierarchical Risk Parity (HRP) method, the naive strategy, and the Ibovespa index, used as a benchmark.

Furthermore, the portfolio constructed using the Markowitz model generated a higher accumulated return when compared to the risk-free rate considered by interbank deposit certificates - CDI or DI, showing how investments in commodity derivatives can be investment options.

These findings are useful for the scientific literature that investigates finance and investments by bringing empirical evidence for Brazilian commodity derivatives to portfolio managers, investors, and companies operating in the supply chain. As a suggestion for future research, different machine learning techniques could be compared for the pre-selection of assets that will make up an investment portfolio in commodity derivatives.

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